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ABSTRACT

This study investigates the extent to which the higher order factors of symbolic and semantic content are differentiated. The main concern is an expository description of the use of confirmatory factor analysis in establishing factorial validity. Reanalyzed data originally collected in 1968 involved a sample of 197 tenth, eleventh and twelfth grade gifted students from a middle-class urban school in Los Angeles. Confirmatory maximum likelihood factor analysis with the LISREL V computer program was used to test the goodness-of-fit to the data of three alternative models of higher order intellectual ability factors: a model of two higher order factors of symbolic and semantic content; a model of one general factor; and a null model of no factor structure. Three different matrices were specified: lambda pattern, phi, and a theta delta. Five procedures for assessing the fit of the proposed models included: (1) examination of LISREL V analysis results; (2) goodness-of-fit index; (3) root mean square residual; (4) Tucker-Lewis reliability coefficient, rho; and (5) a comparison of nested models. The two factor model appears to provide a better data fit than both the one factor or null model. The hypothesis that a previous differential correlation was inflated by transformations tests was supported. (PN)

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THE CONSTRUCT VALIDITY OF HIGHER ORDER FACTORS EMPHASIZING SYMBOLIC AND SEMANTIC CONTENT ABILITIES

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Confirmatory factor analysis is a relatively new statistical procedure which has yet to make a significant breakthrough in educational research. At the same time the use of conventional exploratory factor analysis is not only widespread but also escalating. However, Cronbach (1971) has warned that conventional analysis is a "treacherous technique, largely because it can arrange the data in many ways" (p. 470). For instance, traditionally, researchers have employed somewhat arbitrary methods for estimating communalities and for rotating axes. Such methods have provided results lacking in uniformity and replicability.

In contrast to conventional exploratory factor analysis, Joreskog's (1969) confirmatory factor maximum likelihood factor analysis overcomes the above problems by providing a statistical test of goodness-of-fit for not only the respective structural models generated by conventional factor analysis in the context of a new data base, but also alternative models representing hypotheses derived from an investigator theory. Therefore, the researcher can determine which structural model best explains the causal network responsible for the observed data, namely, the observed pattern of the intercorrelationships among measured variables. Because of this characteristic confirmatory factor analysis holds much promise for future research. Although the potential of confirmatory maximum likelihood is widely recognized (Joreskog, 1969, Nunnally, 1978), there are currently few applications in the literature. Furthermore, many of these applications (e.g., Kenney, 1976; Lomax & Algina, 1979, Marsh & Hocevar, 1981; Schmitt, 1978) focus on the confirmatory factor analysis of

multitrait-multimethod matrices rather than address the more general issues that arise when one is trying to determine the factor structure of a given measurement. This kind of the factorial validity is the major concern of the present investigation.

In a previous analysis, Khattab, Michael and Hocevar (1982) found evidence for a third order factor of symbolic and semantic content in a correlation matrix of 46 structure-of-intellect (SOI) tests emphasizing the product of the transformation. However, a certain amount of built-in bias was found in the Khattab et al. (1982) analysis. The bias occured because of the presence of the large number of tests (32 out of 46) designed to reveal the product of transformations which very possibly created inflated levels of intercorrelations among SOI tests that prevented the emergence of more distinct higher order dimensions of symbolic and semantic content. (The correlation between these two higher order latent factors was .615.) Furthermore, the predominance of transformation measures may have also inflated the factor loadings that were reported in the initial analysis.

A reasonable solution in such a case is to control for the effect of the predominance of transformations by removing some of the SOI tests which reflect the product of transformations while including in the analysis a variety of SOI tests which do not over-represent any single SOI dimension.

Purpose

The purpose of the present study is to further investigate the extent to which the higher order factors of symbolic and semantic content are differentiated. In addition, the present study is an expository description of the use of confirmatory factor analysis in establishing factorial validity.

Methods and Data Source

Data Source

Data from the Aptitudes Research Project at the University of Southern California were reanalyzed. The original data was a correlation matrix of 46 SOI tests reported by Hoepfner et al. (1968), using a sample of 197 tenth-, eleventh- and twelfth-grade subjects in a gifted program of a middle-class urban school in Los Angeles.

Methods

In order to control for the effect of the product of transformation on the intercorrelation of the higher order factors, a new correlation matrix of 18 SOI tests was constructed. In this new correlation matrix a careful balance among the SOI tests that represent different higher order factors across the three facets of the SOI model was achieved. The SOI tests and their corresponding numbers in the original correlation matrix which were selected to serve the purpose of the present investigation are shown in Tables 1 and 2.

In the present study, Joreskog's (1969) confirmatory maximum likelihood factor analysis with the LISREL V computer program (Joreskog and Sorbom, 1981) was used to test the goodness-of-fit to the data of three alternative models of higher order

intellectual ability factors. Specifically, these models included: (a) a model of two higher order factors of symbolic and semantic content. There is empirical support for such a model (Guilford, 1980; Khattab, Michael & Hocevar, 1982); (b) a model of one general factor; and (c) a null model of no factor structure.

Models Specifications

Prior to the computational procedures of each of the proposed models, three different matrices were specified. The Lambda pattern matrix which identifies which test variable is free to be estimated and which one is fixed (given value of 0) for each factor, was established for planing purposes (Tables 1 and 2. In each matrix, a column stood for a hypothesized higher order factor, and each row represented a test variable. The phi matrix (i.e. factor correlations) specifies which factor correlations are free to vary and which are not free to vary.

In the two factor model, all loadings for the two factors were estimated by LISREL V. A single factor correlations was also estimated. Finally, since the correlation of factors with themselves cannot vary from unity, these values were fixed at unity. In the one factor model, the phi matrix was set at unity, a value which represents the correlation between the factor and itself. Subsequently, a final matrix which must be specified prior to the testing procedures is the theta delta matrix. Estimates the error/uniqueness of each variable are given in this matrix. In the present investigation, the theta delta matrix was specified as a diagonal one in which all elements of the diagonal

were set free and estimated, whereas all off-diagonal elements were fixed at zero.

Assessment of Fit of the Proposed Models.

The first and most obvious procedure for assessing the goodness-of-fit of the hypothesized model as a plausible causal structure which generated the observed data is to examine the results of the LISREL V computer analysis. Joreskog and Sorbom (1981) have recommended that the investigator should pay careful attention to the following quantities: (a) parameter estimates, (b) standard errors, (c) squared multiple correlations, (d) coefficients of determination, and (e) correlations between parameter estimates. They have indicated also, that if any of the above quantities has an unreasonable value, this is an indication that the model is fundamentally wrong and that it is not suitable for the data. Unreasonable values might be that some of the parameter estimates are negative, correlations are larger than one in magnitude, and standard errors are extremely large in magnitude. Other indications of a bad model are squared multiple correlation (a measure of the strength of relationship) or coefficients of determination (a measure of the strength of several relationships jointly) which are negative. Large positive values of these measures show how well the observed variables, separately or jointly, serve as measurements for the latent variables. The value of these coefficients are between zero and one; the large value of such coefficients are associated with a good model. Joreskog and Sorbom (1981) have also indicated that the overall fit of the structural equation model may be evaluated by three measures of goodness of fit. One is

the overall X^2 measure and its associated degree of freedom. X^2 is the likelihood ratio test statistic for testing whether the data are congruent with the hypothesized model (Joreskog, 1977, 1981). Large value for X^2 in relation to the number of degrees of freedom suggest a poor fit between the observed data and the hypothesized model. In other words, substantial discrepancies occur between the observed data and those that implied by the hypothesized model; such a situation can be due to (a) smallness of the sample, (b) an inappropriate number of factors, (c) an untenable hypothesized factor structure, or (d) a combination of these circumstances. Appropriate corrective actions based on theoretical and statistical considerations would lead to a smaller X^2 value relative to the number of degrees of freedom and thus to an improved representation of the purported causal process in the structural model - that is, a more reasonable factor structure than that formerly found in the initial model in terms of accounting for the variance and covariance underlying the observed correlation matrix.

Other indices for evaluating the overall fit of structural equation models are the goodness-of-fit index (GFI) - a measure of the relative amount of variance and covariance jointly accounted for by the model, and the root mean square residual (RMR) - a measure of the average of the residual variances and covariances. The RMR measure can be used to compare the fit of two different structural equation models for the same data. The goodness-of fit index (GFI) can also be used for this purpose. In addition, the GFI can be used to compare the fit of structural

equation models for different data bases. Both of these indices should be between zero and one.

A fourth procedure for evaluating the closeness of fit is the Tucker-Lewis (Tucker & Lewis, 1973) reliability coefficient, rho, (the ratio of the amount of covariation explained by a proposed model to the total amount of available covariation to be accounted for by a model). This reliability coefficient takes the following computational form:

$$rho = P_o - P_m \; / \; (P_o - 1)$$

where P_o is the X^2/df ratio for a null model and P_m is X^2/df ratio for the tested model. Tucker and Lewis suggested that a value greater than .90 provides an adequate fit to the data.

A fifth procedure for evaluating the degree of fit of an initially proposed model in relation to an alternative one involves taking the difference in X^2 values between the two models. This difference also is approximately distributed as X^2 with number of degrees of freedom equal to the difference between the numbers of the degree of freedom for the two models. This circumstance is true only if one can specify a less restrictive alternative model that is a subset of the initially proposed model. This kind of comparison is referred to as a comparison of nested models (Bentler, 1980; Joreskog & Goldberger, 1975; Kenny, 1976; Wolfle and Robertshaw, 1981). A large X^2 value from one model to another in relation to the difference in the number of degrees of freedom suggests that the change made in the initial model represent a real improvement in fit. However, if the decrease in X^2 value is approximately equal to the difference in the number of degrees of freedom, then this outcome would suggest

that the change resulted in a minimal improvement in the model because of a chance occurance (Lomax, 1981; Wolfle & Robertshaw, 1981).

Results and Discussion

The steps outlined above were carried out using LISREL V computer program in order to test the fit of the three proposed models to the correlational data in the present investigation. The data analysis yielded the following results.

1. For the higher order model of symbolic and semantic content the estimated factor loadings were found to be statistically significant ($p < .01$). Loadings and their corresponding t-ratios shown in Tables 3 and 4. The estimated correlation between the higher order factors of symbolic and semantic content as presented in Table 5 was found to be .37.

The measures of goodness-of-fit for the whole model as indicated by chi square ratio (X^2/df), the goodness-of-fit index (GFI), the adjusted goodness-of-fit index (AGFI) and the root mean square residual (RMR) were found to be 2.930, .781, .720, and .105, respectively (see Table 8). In addition the Tucker-Lewis index of fit was computed and was found to be .897. As indicated earlier LISREL V also provides a generalized measure of the reliability of the whole measurement model (i.e. coefficient of determination). Such a measure shows how well all the X-variables (the SOI test in the present investigation) jointly serve as measurements for the two higher order factors (i.e. symbolic and semantic content) jointly.

2. For the one factor model all the estimated factor loadings and their corresponding t-ratios (see Tables 6 and 7)

were also found to be statistically significant, ($p < .01$). The phi table for the one factor model is not presented because the correlation of the one latent factor with itself was set at unity. The measures of the goodness-of-fit for the whole model as indicated by chi square ratio (X^2/df), the goodness-of-fit index (GFI), the adjusted goodness-of-fit index (AGFI), the root mean square residual (RMR), and the Tucker-Lewis index (rho) were found to be 3.699, .730, .658, .116, and .856 respectively. A comparison of these values with that of the two factor model indicated that the two factor model provides a better fit to the observed data than the one factor model. In addition, when the two factor model was statistically compared to the one factor model, the two factor model was clearly better tha the one factor model, $X^2 = 106.85$ ($df = 1$).

3. In reference to the null model, the measures of the goodness-of-fit for the whole model as indicated by the chi square ratio (X^2/df), the goodness-of-fit index (GFI), the adjusted goodness-of-fit index (AGFI), and root mean square (RMR) were found to be 19.867, .617, .507, .263, respectively. As one would expect, both the one factor an two factor models were clearly superior to the null model.

In conclusion, a comparison of the two factor model and the one factor model indicated that the two factor model provides a better fit to the data than both the one factor model and the null model. Also, a comparison of the degree of correlation between the higher order factors of symbolic and semantic content obtained by Khattab, Hocevar & Michael (1982) - a value of

approximately .62 - and the value obtained in the present study, .37, clearly supports the hypothesis that the Khattab et al. correlation was spuriously inflated by the presence of numerous transformations tests in the original analysis. Furthermore, both the factor loadings and the factor correlation support the notion that third order factors exist in the Guilford SOI model.

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Table 1

Lambda pattern matrix for the two factor model

	KSI 1	KSI 2
DMC01	0	1
CMI02	0	2
DMT05	0	3
MMT10	0	4
MMT14	0	5
CMI30	0	6
CMU42	0	7
DMT44	0	8
CMU45	0	9
CSU06	10	0
CSU09	11	0
NST11	12	0
DST23	13	0
DSI24	14	0
MSI27	15	0
CST33	16	0
CST37	17	0
DSI40	18	0

Note. Numbers stand for the parameters which were estimated.

Table 2

Lambda pattern for the one factor model

	KSI 1
DMC01	1
CMI02	2
DMT05	3
MMT10	4
MMI14	5
CMI30	6
CMU42	7
DMT44	8
CMU45	9
CSU06	10
CSU09	11
NST11	12
DST23	13
DSI24	14
MSI27	15
CST33	16
CST37	17
DSI40	18

Table 3

Factor Loadings: Two Factor Model

	KSI 1	KSI 2
DMC01	0.0	0.586
CMI02	0.0	0.468
DMT05	0.0	0.585
MMT10	0.0	0.482
CMI30	0.0	0.343
CMU42	0.0	0.500
DMT44	0.0	0.603
CMU45	0.0	0.508
CSU06	0.367	0.0
CSU09	0.577	0.0
NST11	0.645	0.0
DST23	0.598	0.0
DSI24	0.462	0.0
MSI27	0.398	0.0
CST33	0.419	0.0
CST37	0.611	0.0

Table 4

t-ratios for Factor Loadings: Two Factor Model

	KSI 1	KSI 2
DMC01	0.0	7.986
CMI02	0.0	6.164
DMT05	0.0	7.983
MMT10	0.0	6.378
MMT14	0.0	4.393
CMI30	0.0	6.642
CMU42	0.0	8.276
DMT44	0.0	6.762
CMU45	0.0	8.072
CSU06	4.687	0.0
CSU09	7.762	0.0
NST11	8.864	0.0
DST23	8.106	0.0
DSI24	6.031	0.0
MSI27	5.121	0.0
CST33	5.411	0.0
CST37	8.309	0.0
DSI40	3.851	0.0

Table 5

Factor Correlations: Two Factor Model

	PHI	
	KSI 1	KSI 2
KSI 1	1.000	
KSI 2	0.377	1.000

Table 6

Factor Loadings: One Factor Model

	KSI 1
DMC01	0.441
CMI02	0.342
DMT05	0.401
MMT10	0.543
MMT14	0.582
CMI30	0.244
CMU42	0.469
DMT44	0.344
CMU45	0.429
CSU06	0.302
CSU09	0.399
NST11	0.588
DST23	0.545
DSI24	0.496
MSI27	0.315
CST33	0.291
CST37	0.581
DSI40	0.332

Table 7

t-ratios for Factor Loadings: One Factor Model

	KSI 1
DMC01	5.898
CMI02	4.483
DMT05	5.322
MMT10	7.486
MMT14	8.129
CMI30	3.142
CMU42	6.320
DMT44	4.503
CMU45	5.735
CSU06	3.934
CSU09	5.294
NST11	8.238
DST23	7.520
DSI24	6.738
MSI27	4.111
CST33	3.782
CST37	8.112
DSI40	4.344

Table 8

Goodness-of-Fit Results for Contrasting Factor Models

	X^2	df	X^2/df	Tucker-Lewis index rho
Two factor model	392.63	134	2.930	.897
One factor model	499.48	135	3.699	.856
Null model	556.28	28	19867	